PHYSICAL REVIEW B **77**, 113309 (2008)

Effects of growth-direction electric and magnetic fields on excitons in GaAs-Ga$_{1-x}$Al$_x$As coupled double quantum wells

A. L. Morales,[1] N. Raigoza,[1] C. A. Duque,[1,2] and L. E. Oliveira[2]
[1]*Instituto de Física, Universidad de Antioquia, AA 1226, Medellín, Colombia*
[2]*Instituto de Física, Unicamp, Caixa Postal 6165, Campinas, São Paulo 13083-970, Brazil*

(Received 21 December 2007; revised manuscript received 1 February 2008; published 17 March 2008)

Direct and indirect excitons in GaAs-Ga$_{1-x}$Al$_x$As coupled double quantum wells, under growth-direction applied electric and magnetic fields, have been theoretically investigated within a variational procedure in the effective-mass and parabolic-band approximations. The exciton hydrogenic 1*s*-like envelope wave function is obtained through a variational procedure and an appropriate expansion in trigonometric functions of the electron and hole wave functions. The applied electric field produces a polarization of the exciton by pushing the electron and hole away from each other, whereas the magnetic field contracts the exciton by pushing the electron and hole closer to each other. Intersubband mixing produced by the Coulomb interaction of electron-hole pairs is taken into account and a detailed analysis of the properties of direct- and indirect-exciton states in GaAs-Ga$_{1-x}$Al$_x$As coupled double quantum wells is presented, with theoretical results in good agreement with available experimental measurements.

DOI: 10.1103/PhysRevB.77.113309 PACS number(s): 71.55.Eq, 73.20.Mf, 73.21.Fg

Coupled double GaAs-Ga$_{1-x}$Al$_x$As quantum well (CDQW) structures under applied electric, magnetic, and stress fields have attracted much attention recently. Some examples of phenomena studied on these systems are the excitonic Bose–Einstein condensation, two dimensional (2D) electron gas properties, optical absorption, reflection and photoluminescence (PL) of direct and indirect excitons, donors and acceptors, properties of charged excitons, transition between exciton and 2D electron gas or magnetoexciton regimes, exciton trapping, quantum Stark effect, magnetoexciton dispersion relations, electron-phonon interaction, electron transport, etc. The wide range of applications also attracts a great interest in spintronics, data storage, electronic and optoelectronic devices, lasers, and terahertz detectors.[1–3]

When an electric field is applied along the growth direction of a symmetrical thin-barrier GaAs-Ga$_{1-x}$Al$_x$As doubled quantum well (DQW), the exciton energy decreases as the field increases and the ground-state exciton may be formed with the electron and hole in different wells, i.e., the exciton ground state may change from a direct-exciton to an indirect-exciton state. On the other hand, if a magnetic field is applied along the growth direction of a GaAs-Ga$_{1-x}$Al$_x$As quantum well (QW), an increase in exciton binding energy is found as the magnitude of the magnetic field increases, due to the shrinking of the exciton wave function. The anticrossing between the ground and first-excited exciton states in GaAs-Ga$_{1-x}$Al$_x$As CDQWs has been thoroughly studied experimentally.[1–4] The simultaneous application of an electric field in the growth direction and a magnetic field parallel to the interfaces of the CDQW produces strong changes in the PL spectra and kinetics of the indirect (interwell) excitons due to the magnetic field induced displacement of the indirect exciton dispersion in momentum space[3,4] which may be used to increase the exciton lifetime. In a recent theoretical work, de Dios-Leyva *et al.*[5] have studied the effects of crossed growth-direction applied electric and in-plane magnetic fields on the exciton direct and indirect states in GaAs-Ga$_{1-x}$Al$_x$As CDQWs and found good agreement with available PL experimental measurements.[3,4]

In the present work, we are, in particular, concerned with a proper theoretical understanding of the experimental findings by Butov *et al.*[4] in GaAs-Ga$_{1-x}$Al$_x$As CDQWs under electric and magnetic fields applied simultaneously in the growth direction (z direction) of the heterostructure. Here, theoretical calculations are performed by using a cylindrical gauge $\vec{\mathbf{A}}=\frac{1}{2}(\vec{\mathbf{B}}\times\vec{\mathbf{r}})$ and envelope-function and parabolic-band approximations. The Hamiltonian for the exciton then may be written as[5,6]

$$\hat{H} = \frac{1}{2m_e^*}\left[\hat{\vec{p}}_e + \frac{e}{c}\vec{A}(\vec{r}_e)\right]^2 + \frac{1}{2m_h^*}\left[\hat{\vec{p}}_h - \frac{e}{c}\vec{A}(\vec{r}_h)\right]^2 + V_e(z_e) + V_h(z_h) - \frac{e^2}{\epsilon|\vec{r}|} + e\vec{E}\cdot\vec{r}, \qquad (1)$$

where $\vec{r}=\vec{r}_e-\vec{r}_h$ is the e-h relative coordinate, $\hat{\vec{p}}_\alpha$, $\vec{r}_\alpha$, m_α^*, and V_α, with $\alpha=e,h$, are the momentum operators, electron and hole coordinates, effective masses, and corresponding CDQW confining potentials, respectively, e is the absolute value of the electron charge, and ϵ is the GaAs dielectric constant [low-temperature material parameters[7] used in this work are as follows: $m_e^*/m_0=0.0665$, $m_h^*/m_0=0.34$ (m_0 is the free electron mass), and $\varepsilon=12.4$]. The eigenfunctions of the exciton Hamiltonian are written as $\Psi_{exc}(\vec{r}_e,\vec{r}_h)=\exp\left(\frac{i}{\hbar}\vec{P}\cdot\vec{R}\right)/\sqrt{S}\Phi(\vec{\rho},z_e,z_h)$, where the exciton conserved momentum is $\vec{P}=(P_x,P_y)$, S is the CDQW transverse area, the in-plane exciton center-of-mass and relative coordinates are $\vec{R}$ and $\vec{\rho}$, respectively, and $\Phi(\vec{\rho},z_e,z_h)$ is the eigenfunction (with E_X exciton energy) of

$$\hat{H} = \frac{P^2}{2M} + \frac{\hat{p}_\rho^2}{2\mu} + \hat{H}_e + \hat{H}_h + \gamma L_z + \frac{\gamma^2}{4}\rho^2 - \frac{e^2}{\epsilon|\vec{r}|}, \qquad (2)$$

with

©2008 The American Physical Society

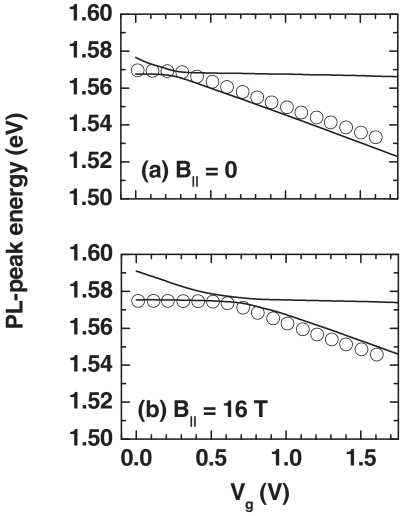

FIG. 1. PL-peak transition energy as a function of the applied gate voltage for a GaAs-Ga$_{0.67}$Al$_{0.33}$As symmetric CDQW of dimensions $L_W=80$ Å, $L_B=40$ Å, under growth-direction applied electric and magnetic fields. Open dots are the corresponding experimental data of Butov *et al.* (Ref. 4).

$$\hat{H}_\alpha = \frac{-\hbar^2}{2m_\alpha^*}\frac{\partial^2}{\partial z_\alpha^2} + V_\alpha(z_\alpha) + \sigma_\alpha eEz_\alpha, \qquad (3)$$

where $\hat{\vec{p}}_\rho = -i\hbar\frac{\partial}{\partial\vec{\rho}}$, the total and reduced exciton masses are M and μ, respectively, $\gamma=\frac{e\hbar B}{2\mu cR}$ (R is the effective Rydberg), $\sigma_e=-1$, $\sigma_h=+1$, and the applied electric field $\vec{E}=-E\hat{z}$.

The exciton variational[5] wave functions of the GaAs-Ga$_{1-x}$Al$_x$As CDQW may be written as $\Phi(\vec{\rho},z_e,z_h) = f(z_e)F(z_h)e^{-\lambda r}$, where $r=\sqrt{\rho^2+(z_e-z_h)^2}$, λ is a variational parameter, and $f(z_e)$ and $F(z_h)$ are, in general, linear combinations of the z-dependent part of the electron $f_i(z_e)$ and hole $F_j(z_h)$ eigenfunctions of the Hamiltonian without the Coulomb interaction. We note that the noncorrelated $f_i(z_e)$ and $F_j(z_h)$, which are adapted to the situation with an applied electric field, are readily obtained via the method by Xia and Fan[8] of expansion in terms of sine functions. At low temperatures, one may take $\vec{k}=0$ and $\vec{P}=0$. Here, we follow the variational procedure outlined by de Dios-Leyva *et al.*[5] and only take into account the mixing between the DQW electron $f_0(z_e)$ ground state and $f_1(z_e)$ first-excited state.

Figures 1(a) and 1(b) show the exciton PL-peak energy as a function of the applied gate voltage (or electric field; 23.8 kV/cm corresponds to 1 V) for a GaAs-Ga$_{0.67}$Al$_{0.33}$As symmetric CDQW of dimensions well $L_W=80$ Å, barrier $L_B=40$ Å. The solid curves in Figs. 1(a) and 1(b) show the theoretical exciton PL-peak energies, corresponding to the ground state and first-excited exciton state, for $B=0$ [Fig. 1(a)] and growth-direction applied $B=16$ T [Fig. 1(b)], respectively, whereas the open dots correspond to the experimental findings of Butov *et al.*[4] The open dot size corresponds essentially to the experimental error bars. The experimental data show a constant exciton PL-peak energy behavior at low values of the electric fields corresponding to an exciton ground state associated with the spatially direct-exciton regime, in which both the electron and hole remain in the same well. When the electric field is switched on, in the $-z$ direction, its effect pushes the electron toward the neighbor well and pulls the hole away from the electron, with the result that at a certain strength of the applied electric field, the transition to the exciton ground state corresponding to a spatially indirect-exciton regime occurs. Here, the electron tunnels to the other well and the hole is pushed the farthest from the electron. In this regime, the ground-state indirect-exciton results in a PL-peak energy with a decreasing linear behavior as a function of the increasing applied electric field, with the gate-voltage value at which the direct to indirect exciton transition occurs increasing from 0.3 V at $B=0$ to 0.6 V at $B=16$ T. Also, one notes that, for a GaAs-Ga$_{0.67}$Al$_{0.33}$As symmetric CDQW of dimensions $L_W=80$ Å, $L_B=40$ Å, the excited exciton state, corresponding to a spatially direct exciton, appears as a PL-peak energy which remains essentially constant with increasing values of the applied electric field. The ground-state theoretical results are in quite good agreement with the exciton PL peaks in the experimental findings by Butov *et al.*[4] One notices that the ground-state exciton PL peak diminishes in energy as the growth-direction applied electric field increases, for fixed values of the applied magnetic field. Moreover, the ground-state exciton PL peak increases in energy as the growth-direction applied magnetic field increases, for fixed values of the applied gate voltage. This is due to the interplay between the polarization caused by the electric field, effect of the Coulomb interaction, and effects of the central and edge barriers. The electric field produces a spatial displacement of the electron and hole in opposite directions, whereas the electron-hole Coulomb force develops a component along the z direction which opposes the external electric field and diminishes its effect, whereas the barriers constrain the electron and hole movement.

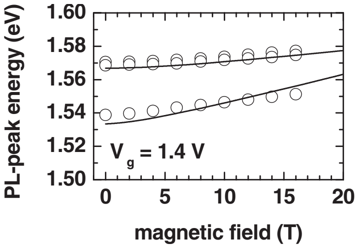

FIG. 2. Applied magnetic-field dependence, at a fixed applied $V_g=1.4$ V gate voltage, of the PL-peak energy corresponding to direct (higher curve) and indirect (lower curve) excitons for a GaAs-Ga$_{0.67}$Al$_{0.33}$As CDQW of dimensions $L_W=80$ Å, $L_B=40$ Å. Open symbols are the corresponding experimental data from Butov *et al.* (Ref. 4).

Figure 2 shows a comparison of the present theoretical calculations with the PL-peak experimental (open dots) results by Butov *et al.*[4] for the indirect- (lower curve) and direct-exciton (upper curve) energies as functions of the growth-direction applied magnetic field at a fixed applied gate voltage $V_g=1.4$ V for a GaAs-Ga$_{0.67}$Al$_{0.33}$As symmetric CDQW of dimensions $L_W=80$ Å, $L_B=40$ Å. At this gate-voltage value, the ground and first-excited states are away from the resonance region. The lower theoretical curve was obtained by using a bonding exciton variational wave function, and the upper curve corresponds to states described by an antibonding variational wave function [cf. Eqs. (11) and (12) by de Dios-Leyva *et al.*[5]]. It is apparent that results obtained by the present theoretical model follow closely the experimental findings for the direct-exciton case. In the case of the indirect exciton (lower curve in Fig. 2), the slope of the calculated magnetic-field dependence of the indirect exciton dispersion, for a $V_g = 1.4$ V gate voltage, is clearly larger than for the experimental results. It can be seen that the exciton PL-peak energies increase as the applied magnetic field increases, with the increment being larger for the case of the indirect-exciton PL peak. The differences with respect to the experimental values may come from minor changes in the measured well widths, electron, and hole effective-mass dependence with the magnetic field, nonparabolicity effects, the fact that the present model ignores valence-band mixing, etc. Also, as the magnetic field has the effect of shrinking the exciton wave function in the xy plane while the electric field tends to separate the electron and hole along the z direction, a two-variational parameter wave function, in which the z and ρ coordinates would be free to adapt to each specific situation, may provide a more realistic description of the experimental data. Moreover, from Fig. 5, in the study by Butov *et al.*,[4] it is clear that there is an energy shift in the indirect-exciton regime, with applied magnetic field, and it strongly depends on the excitation density. As pointed out by Butov *et al.*,[4] the indirect-exciton PL energy is found to enhance with density, which reflects the net repulsive interaction between indirect excitons. Of course, such an effect is not considered in the present correlated *e-h* theoretical model. In addition, with respect to the double experimental dots corresponding to the upper curve of Fig. 2, we believe that further theoretical and experimental works are needed in order to quantitatively understand the PL higher-energy double-peaked structure found in the experimental measurements by Butov *et al.*,[4] who pointed out that it is likely a result of difference in widths of the two nominally identical GaAs QWs.

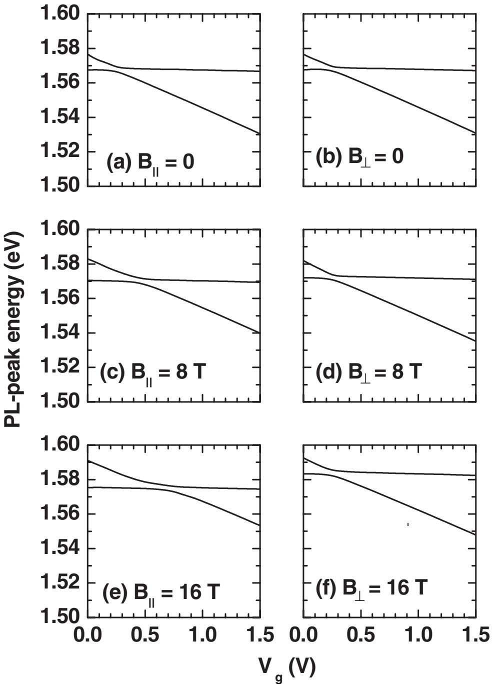

FIG. 3. Comparison of PL-peak transition energies for a GaAs-Ga$_{0.67}$Al$_{0.33}$As CDQW of dimensions $L_W = 80$ Å, $L_B = 40$ Å under the effects of applied electric and magnetic fields. (a), (c), and (e) are for growth-direction applied electric and magnetic fields, whereas (b), (d), and (f) are for an in-plane applied magnetic field (perpendicular to the growth direction) and growth-direction applied electric field (Ref. 5).

Figure 3 presents a comparison of the calculated direct- and indirect-exciton PL-peak energies for a GaAs-Ga$_{0.67}$Al$_{0.33}$As CDQW of dimensions $L_W = 80$ Å, $L_B = 40$ Å under the simultaneous effects of growth-direction applied electric and magnetic fields [Figs. 3(a), 3(c), and 3(e)], with results obtained for the CDQW under in-plane applied magnetic field[5] and growth-direction applied electric field [Figs. 3(b), 3(d), and 3(f)]. It is apparent from Fig. 3 that the gate voltage, corresponding to the direct-indirect anticrossing, moves to higher values as the growth-direction applied magnetic field increases, whereas for in-plane applied magnetic fields, the anticrossings remain at essentially the same gate-voltage value. Here, we should mention that, in a previous study,[5] it was shown that for an asymmetric GaAs-Ga$_{1-x}$Al$_x$As CDQW such crossings present a slight dependence on the in-plane applied magnetic field and, therefore, this dependence seems to be related to the particular geometry of the CDQW. The shift to higher applied electric fields of the crossover when the growth-direction magnetic field increases may be understood as follows:[4] In the direct-exciton case, the *e-h* distance is essentially associated with the ρ distance between the carriers, whereas in the indirect case, the *e-h* distance, mainly depends on the z separation, which in the indirect-exciton regime is essentially constant and equal to the separation between the centers of the two QWs. Changes in the exciton binding energy with the applied magnetic field are associated with changes in the *e-h* distance, which affect the Coulomb interaction. Therefore, as the magnetic field is applied in the growth direction, its effects are mainly seen on the ρ coordinate, and when the magnetic field increases, the *e-h* distance diminishes for the direct-exciton case and remains essentially constant for the indirect-exciton one, leading as a result that the difference between the binding energies for excitons in the direct and indirect regimes increases with the applied growth-direction magnetic field. Due to the fact that the electric-field value in which the crossover appears depends on this difference of binding energies, the crossover must shift to higher gate voltages when the magnetic field increases.

In conclusion, a theoretical study of the direct and indirect magnetoexciton states in GaAs-Ga$_{1-x}$Al$_x$As CDQWs under growth-direction applied magnetic and electric fields is presented. The effective-mass and parabolic-band approximations are used and inter-subband mixing, brought about by the Coulomb interaction of electron-hole pairs in CDQWs, is considered. The exciton envelope wave function is obtained by using a variational procedure with a hydrogenic 1s-like wave function and an expansion in trigonometric functions for the electron and hole wave functions. The experimental anticrossings, i.e., the transition from the intrawell and/or direct- to the interwell and/or indirect-exciton regime, are well reproduced within the present model calculation. The general behavior of the exciton PL-peak energy as a function of the electric field is well reproduced, both for $B=0$ and an applied growth-direction magnetic field of $B=16$ T. Away from the anticrossing regions, at a somewhat high electric-field value, the theory still reproduces the experimental relation between the exciton PL-peak energy and applied magnetic field. It may be seen that the exciton PL-peak energies increase for both directions of the applied magnetic field. The difference in exciton behavior when subjected either to growth-direction or in-plane magnetic fields comes from the fact that the growth-direction magnetic field compresses the exciton probability density in the ρ direction, whereas the in-plane magnetic field shrinks it along the z direction.

ACKNOWLEDGMENTS

The authors would like to thank the Colombian CODI-Universidad de Antioquia and Brazilian Agencies CNPq, FAPESP, MCT-Millenium Institute for Quantum Information, and MCT-Millenium Institute for Nanotechnology for partial financial support. This work was also partially financed by the Excellence Center for Novel Materials and COLCIENCIAS under Contract No. 043-2005. A.L.M. and N.R. wish to acknowledge the warm hospitality of the Instituto de Física of UNICAMP, where part of this work was performed.

---

[1] M. Orlita, R. Grill, P. Hlídek, M. Zvára, G. H. Döhler, S. Malzer, and M. Byszewski, Phys. Rev. B **72**, 165314 (2005); B. M. Ashkinadze, E. Cohen, V. V. Rudenkov, P. C. M. Christianen, J. C. Maan, and L. N. Pfeiffer, *ibid.* **76**, 075344 (2007); A. Gärtner, L. Prechtel, D. Schuh, A. W. Holleitner, and J. P. Kotthaus, *ibid.* **76**, 085304 (2007); Kyu-Seok Lee, Sam Kyu Noh, and Soo Kyung Chang, *ibid.* **76**, 073305 (2007), and references therein.

[2] I. Galbraith and G. Duggan, Phys. Rev. B **40**, 5515 (1989); R. L. Greene and K. K. Bajaj, *ibid.* **37**, 4604 (1988); A. M. Fox, D. A. B. Miller, G. Livescu, J. E. Cunningham, and W. Y. Jan, *ibid.* **44**, 6231 (1991); J. Cen, S. V. Branis, and K. K. Bajaj, *ibid.* **44**, 12848 (1991); J. Soubusta, R. Grill, P. Hlídek, M. Zvára, L. Smrcka, S. Malzer, W. Geißelbrecht, and G. H. Döhler, *ibid.* **60**, 7740 (1999); Yu. E. Lozovik, I. V. Ovchinnikov, S. Yu. Volkov, L. V. Butov, and D. S. Chemla, *ibid.* **65**, 235304 (2002).

[3] L. V. Butov, A. Imamoglu, A. V. Mintsev, K. L. Campman, and A. C. Gossard, Phys. Rev. B **59**, 1625 (1999); L. V. Butov, A. V. Mintsev, Yu. E. Lozovik, K. L. Campman, and A. C. Gossard, *ibid.* **62**, 1548 (2000); A. Parlangeli, P. C. M. Christianen, J. C. Maan, I. V. Tokatly, C. B. Soerensen, and P. E. Lindelof, *ibid.* **62**, 15323 (2000).

[4] L. V. Butov, A. A. Shashkin, V. T. Dolgopolov, K. L. Campman, and A. C. Gossard, Phys. Rev. B **60**, 8753 (1999).

[5] M. de Dios-Leyva, C. A. Duque, and L. E. Oliveira, Phys. Rev. B **76**, 075303 (2007).

[6] L. P. Gorkov and I. E. Dzyaloshinskii, Sov. Phys. JETP **26**, 449 (1968); K. Chang and F. M. Peeters, Phys. Rev. B **63**, 153307 (2001).

[7] E. H. Li, Physica E (Amsterdam) **5**, 215 (2000).

[8] J.-B. Xia and W.-J. Fan, Phys. Rev. B **40**, 8508 (1989).